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A Comparative Study of Carrier Frequency Offset (CFO) Estimation Techniques for OFDM Systems

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Abstract: *In this paper, we will examine the effects of Carrier Frequency Offset (CFO) on the OFDM signals. We will also show the deterioration in Bit Error Rate (BER) as well as the loss in Signal to Noise Ratio (SNR) due to the CFO for an OFDM system. The paper also presents comparative analysis between three different CFO estimation methods namely: Cyclic Prefix (CP) based estimation technique, Symbol based estimation method and Pilot Tone based estimation scheme. The comparison is done in terms of Mean Square Error (MSE) by using MATLAB Software.*

Keywords: *Carrier Frequency Offset (CFO), Cyclic Prefix (CP), Inter-carrier Interference (ICI), Orthogonal Frequency Division Multiplexing (OFDM)*

I. Introduction

As high data rate transmission is one of the major challenges in modern wireless communications, there is a substantial need for a higher frequency bandwidth. Meanwhile, with the increase of data rate the distortion of the received signals caused by multipath fading channel becomes a major problem.

Orthogonal Frequency Division Multiplexing (OFDM) gives higher bandwidth efficiency by using the orthogonality principle and overcomes the effect of multipath fading channel by dividing the single high data rate stream into a several low data rate streams. OFDM is a multicarrier transport technology for high data rate communication system [1]. The OFDM concept is based on spreading the high speed data to be transmitted over a large number of low rate carriers. The carriers are orthogonal to each other and frequency spacing between them are created by using the Fast Fourier transform (FFT). OFDM is being used in a number of wired and wireless voice and data applications due to its flexible system architecture. Some examples of current OFDM applications are DAB (digital audio broadcasting), HDSL (High-Rate Digital Subscriber Line), VHDSL (Very High-Rate Digital Subscriber Line), ADSL (Asymmetric DSL), HDTV-Terrestrial (High Definition TV-T), IEEE 802.11 and HiperLAN/2, GSTN (General Switched Telephone Network), Cellular Radio and DVB-T (digital video broadcasting-terrestrial) [2].

However, despite all these advantages, OFDM has some drawbacks including its higher peak to average power ratio and suffers higher sensitivity to Carrier Frequency Offset (CFO). There are two main causes of CFO. The first is a frequency mismatch between the local oscillators at the transmitter and receiver which results in residual CFO at the receiver after the down-conversion process. The second cause is the Doppler shift which is a result of the relative motion between the transmitter and receiver present in mobile environments. CFO is a major drawback in OFDM systems which disturbs the orthoganality among the sub-carriers and consequently introduces Inter-Carrier Interference (ICI). CFO also increases the Bit Error Rate (BER) and degrades SNR of the signal [3].

By estimating the CFO at the receiver, the loss in performance due to the CFO can be significantly reduced. Many methods have been proposed in the literature to estimate and compensate for the CFO. CFO estimation techniques can be broadly classified into Pilot-aided schemes and non-pilot aided (blind) estimation schemes. Pilot assisted methods use well defined pilot symbols to aid in the estimation of CFO. This method is popular and capable of achieving very quick and reliable estimates, though there is a loss in data rate and spectrum efficiency of the system. Blind (non-pilot aided) methods exploit the structural and statistical properties of the transmitted signals. Though these techniques preserve the data rate, they lead to processing the received data to multiple times, which causes delay in decoding [4].

This study will focus on to investigate the effects of CFO on the received OFDM signal and shows the OFDM performance degradation, both in BER and in SNR, caused by the CFO. The paper also compares the performance of three different CFO estimation techniques in terms of their MSE.

II. Ofdm System Model

Fig. 1 shows a simple baseband OFDM transceiver system model.

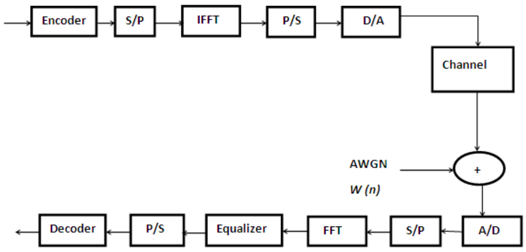

Figure 1. Baseband OFDM System model

In an OFDM system, at the transmitter part, a high data-rate input bit stream b[n] is converted into N parallel bit streams each with symbol period T_s through a serial-to-parallel buffer. When the parallel symbol streams are generated, each stream would be modulated and carried over at different center frequencies. The sub-carriers are spaced by $1/N\,T_s$ in frequency, thus they are orthogonal over the interval $(0,T_s)$. Then, the N symbols are mapped to bins of an Inverse Fast Fourier Transform (IFFT). These IFFT bins correspond to the orthogonal sub-carriers in the OFDM symbol. Therefore, the OFDM symbol can be expressed as

$$X\,(n) = \frac{1}{N}\sum_{m=0}^{N-1} X_m \exp\frac{j2\pi nm}{N} \qquad (1)$$

where the $X(m)$ are the baseband symbols on each sub-carrier. Then, the $X(i)$ points are converted into a time domain sequence $x(i)$ via an IFFT operation and a parallel-to-serial conversion. The digital-to-analog (D/A) converter then creates an analog time-domain signal which is transmitted through the channel. At the receiver, the signal is converted back to a discrete N point sequence $y(n)$, corresponding to each subcarrier. This discrete signal is demodulated using an N-point Fast Fourier Transform (FFT) operation at the receiver. The demodulated symbol stream is given by

$$Y(m) = \sum_{n=0}^{N-1} y(n) \exp\frac{-j2\pi nm}{N} + W(m) \qquad (2)$$

where$w(m)$ corresponds to the FFT of the samples of $w(n)$, which is the time invariant Additive White Gaussian Noise (AWGN) introduced in the channel. Then, the signal is down-converted and transformed into a digital sequence after it passes an Analog-to-Digital Converter (ADC). The following step is to pass the remaining T_D samples through a parallel-to-serial converter and to compute N-point FFT. The resulting $Y(i)$ complex points are the complex baseband representation of the N modulated subcarriers. As the broadband channel has been decomposed into N parallel sub channels, each sub channel needs an equalizer (usually a 1-tap equalizer) in order to compensate the gain and phase introduced by the channel at the sub channel's frequency. These blocks are called Frequency Domain Equalizers (FEQ).Therefore the groups of bits that has been placed on the subcarriers at the transmitter are recovered at the receiver as well as the high data-rate sequence [5].

III. Effects Of Cfo

Let f_c and f_c' denote the carrier frequencies in the transmitter and receiver, respectively. Let f_{offset} denote their difference ($f_{offset} = f_c - f_c'$). Meanwhile, Doppler frequency f_d is determined by the carrier frequency f_c and the velocity v of the terminal (receiver) as [6]

$$f_d = \frac{vf_c}{c} \qquad (3)$$

where$c$ is the speed of light. Let us define the normalized CFO, ε, as a ratio of the CFO to subcarrier spacing Δf, as

$$\varepsilon = \frac{f_{offset}}{\Delta f} \qquad (4)$$

After normalizing the CFO by the subcarrier spacing, the integer part and the fractional part of the CFO can be estimated separately. Estimation of the integer part can be termed as coarse CFO estimation while the estimation of the fractional part of the CFO can be termed as fine estimation of the CFO [4]. Let $\varepsilon_i$and ε_f denote the integer and fractional part ofε, respectively, and therefore, $\varepsilon = \varepsilon_i + \varepsilon_f$.

The integer part of CFO ε_i, causes a cyclic-shift in the receiver, and this result a significant degradation in the BER performance. However, integer CFO does not cause the loss of orthogonality among the subcarrier frequency components and thus, ICI does not occur [6].

The fractional CFO is the part which destroys the orthogonality among the subcarriers and consequently introduces ICI. Let's now mathematically describe the effect of $\varepsilon_f$on the received OFDM signal. Ignoring the effects of the additive noise, the received signal before taking FFT can be expressed as

$$Y_l[n] = \frac{1}{N}\sum_{m=0}^{N-1} H[m]\,X_l[m] \exp\frac{j2\pi n(m+\varepsilon_f)}{N} \qquad (5)$$

After taking FFT of the above equation, we will get the received signal which is affected by fractional CFO, and can be written as

$$Y_l[k] = \sum_{n=0}^{N-1} Y_l[n] \exp\frac{-j2\pi kn}{N} \qquad (6)$$

Putting the value of $Y_l[n]$ in equation (6), we may write

$$Y_l[k] = \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} H[m]X_l[m] \exp\frac{j2\pi n(m+\varepsilon_f)}{N} \exp\frac{-j2\pi kn}{N}$$

$$= \frac{1}{N}\sum_{m=0}^{N-1} H[m]X_l[m] \sum_{n=0}^{N-1} \exp\frac{j2\pi(m-k+\varepsilon_f)}{N}$$

Now, considering the case where $k = m$

$$Y_l[k] = \frac{1}{N} H[k]X_l[k] \sum_{n=0}^{N-1} \exp\frac{j2\pi n\varepsilon_f}{N} + \frac{1}{N}\sum_{\substack{m=0\\ m\neq k}}^{N-1} H[m]X_l[m] \sum_{n=0}^{N-1} \exp\frac{j2\pi(m-k+\varepsilon_f)}{N} \qquad (7)$$

After rearranging and simplifying the above equation, we have got

$$Y_l[k] = \frac{\sin \pi\varepsilon_f}{N \sin\frac{\pi\varepsilon_f}{N}} \exp\frac{j\pi\varepsilon_f}{N} H[k]X_l[k] + I_l[k] \qquad (8)$$

where

$$I_l[k] = \exp\frac{j\pi\varepsilon_f(N-1)}{N} \sum_{\substack{m=0\\ m\neq k}}^{N-1} H[m]X_l[m] \frac{\sin \pi(m-k+\varepsilon_f)}{N \sin\frac{\pi(m-k+\varepsilon_f)}{N}} \exp\frac{j\pi(m-k)(N-1)}{N}$$

The first term of equation (8), $\frac{\sin \pi\varepsilon_f}{N \sin\frac{\pi\varepsilon_f}{N}} \exp\frac{j\pi\varepsilon_f}{N}$, represents the amplitude and phase distortion of thek^{th} subcarrier frequency component due to the fractional CFO. Meanwhile, $I_l[k]$ in equation (8) represents the ICI from the other subcarriers into k^{th} frequency component, which implies that the orthogonality among subcarrier frequency components is not maintained any longer due to the fractional CFO [6].

A. SNR degradation due to CFO

According to [7], the degradation of SNR, D_{freq}, caused by the frequency offset is approximated as

$$D_{freq} \cong \frac{10}{3 \ln 10} (\pi\Delta f T)^2 \frac{E_b}{N_o} \qquad (9)$$

whereΔf is the frequency offset, Tis the symbol duration in seconds, E_b is the energy per bit of the OFDM signal and N_o is the one-sided noise power spectrum density (PSD).

IV. Cfo Estimation Techniques

A. CP based Estimation Method

This method exploits the Cyclic Prefix (CP) of the OFDM symbol to estimate the CFO in time domain. Considering the channel effect is minimal and can be neglected, then the l^{th} OFDM symbol affected by CFO can be written as

$$y_l[n] = x(n) \exp\frac{j2\pi\varepsilon n}{N} \qquad (10)$$

We may now consider the corresponding CP in the OFDM symbol

$$y_l(n+N) = x(n+N) \exp\frac{j2\pi\varepsilon}{N}(n+N) \quad (11)$$

$$y_l(n+N) = x(n) \exp\left[\frac{j2\pi\varepsilon n}{N} + j2\pi\varepsilon\right] \quad (12)$$

From equations (10) and (12), we can find that the phase difference between CP and the OFDM symbol which the victim of CFO is $2\pi\varepsilon$. Therefore, the amount of CFO can be found by the multiplication of OFDM symbol (CFO affected) with the CP and after that taking their phase angle measurements, as show below

$$\tilde{\varepsilon} = \frac{1}{2\pi} \arg\{y_l^*[n] y_l[n+N]\}, n = -1, -2, \ldots - N_g \quad (13)$$

In order to reduce the noise effect, its average can be taken over the samples in a CP interval as:

$$\tilde{\varepsilon} = \frac{1}{2\pi} \arg\left\{\sum_{n=-N_g}^{-1} y_l^*[n] y_l[n+N]\right\}, n = -1, \ldots - N_g \; (14)$$

Since $\mathbf{arg}()$ function if performed by using $\mathbf{tan^{-1}}()$ function which has the range of $[-\pi, \pi]$ in an 2π interval, the above equation can estimate CFO in the range of [-0.5, +0.5]. Therefore, CP results CFO estimation in the range $|\tilde{\varepsilon}| \le 0.5$. Hence, this technique is useful for the estimation of Fractional CFO. The drawback of this technique is that it does not estimate the integer offset.

B. Symbol based Estimation Method

The basis of this technique, proposed by P.H. Moose [8], is that same data frame is repeated and the phase value of each carrier between consecutive symbols is compared.

If two identical training symbols are transmitted consecutively, the corresponding signals with CFO of ε are related with each other as follows

$$y_2[n] = y_1[n] \exp \frac{j2\pi\varepsilon N}{N} \leftrightarrow Y_2[k] = Y_1[k] \exp j2\pi\varepsilon \quad (15)$$

Using the relationship in the above equation, the CFO can be estimated as

$$\tilde{\varepsilon} = \frac{1}{2\pi} \tan^{-1} \frac{\left\{\sum_{k=0}^{N-1} \mathrm{Im}\,[Y_1^*[k] Y_2[k]]\right\}}{\left\{\sum_{k=0}^{N-1} \mathrm{Re}\,[Y_1^*[k] Y_2[k]]\right\}} \quad (16)$$

The range of CFO estimation of this method is also limited to $|\tilde{\varepsilon}| \leq \pi/2\pi = 1/2$. However, the range can be increased by D times by using a training symbol with D repetitive patterns.

C. Pilot Tone based Estimation Technique

This method, proposed by Classen [9], suggests the use of pilot tones to estimate CFO in the frequency domain. Pilot tones can be inserted in the frequency domain and transmitted in every OFDM symbol for CFO tracking. First, two symbols, $y_l[n]$ and$y_{l+D}[n]$, are saved in the memory after synchronization. Then, the signals are transformed into $\{Y_l[k]\}_{k=0}^{N-1}$ and $\{Y_{l+D}[k]\}_{k=0}^{N-1}$ via FFT, from which pilot tones are extracted.

In this process, two different estimation modes for CFO estimation are implemented: acquisition and tracking modes. In the acquisition mode, a large range of CFO including an integer CFO is estimated. In the tracking mode, only fine CFO is estimated. The integer CFO is estimated by

$$\widetilde{\varepsilon_{acq}} = \frac{1}{2\pi T_{sub}} \max\left\{\left| \sum_{j=0}^{L-1} Y_{l+D}[P[j], \varepsilon] Y_l^*[P[j], \varepsilon] X_{l+D}^*[P[j]] X_l[P[j]] \right|\right\} (17)$$

where L, P[j] and $X_l[P[j]]$ denote the number of pilot tones, the location of the j^{th} pilot tone and the pilot tone located at P[j] in the frequency domain at the l^{th} symbol period, respectively. Meanwhile, the fine CFO is estimated by

$$\tilde{\varepsilon}_f = \frac{1}{2\pi Tsub} \max\left\{\left| \sum_{j=0}^{L-1} Y_{l+D}[P[j], \widetilde{\varepsilon_{acq}}] Y_l^*[P[j], \widetilde{\varepsilon_{acq}}] X_{l+D}^*[P[j]] X_l[P[j]]\right\} (18)$$

V. Simulation Results

Equation (9) is simulated in MATLAB software to visualize the effects of CFO in the signal SNR. To compare CFO estimation techniques, three equations are simulated. First, by using equation (14), the phase difference between CP and the corresponding rear part of an OFDM symbol. Second, using equation (16), the phase difference between repetitive symbols and finally using equation (17), estimation based on pilot tones in two consecutive symbols. OFDM parameters used in the simulation are given in the table below.

Table:1 Ofdm Parameters Used In Simulation

Parameter	
Modulation scheme	QPSK
Number of data subcarriers	192
Number of guard-band subcarrier	56
Number of Pilot subcarriers	8

Fig. 2 shows that the received signal is severely distorted in the presence of CFO. Fig. 3 indicates that the SNR deterioration is bigger in higher SNR values signals. Fig. 4 shows that the bit error rate increases with the increase of the frequency offset, and the best performance is obtained when the CFO is zero. The performance is symmetric in the offset about the zero offset point. Fig. 5 reveals that the pilot tone based estimation performs the best because of its lowest MSE. Fig. 5 also shows that the mean squared CFO estimation errors decrease as the SNR of received signal increases.

VI. Conclusion

In this paper, we investigated the effects of CFO in OFDM signals. Both mathematical analysis and simulation results showed that CFO violates the orthogonality principle, and hence introduce ICI, decreases the SNR of the signal and increases the bit error rate. The pilot based estimation technique has the best performance among the other techniques examined in this paper in terms of their MSE.

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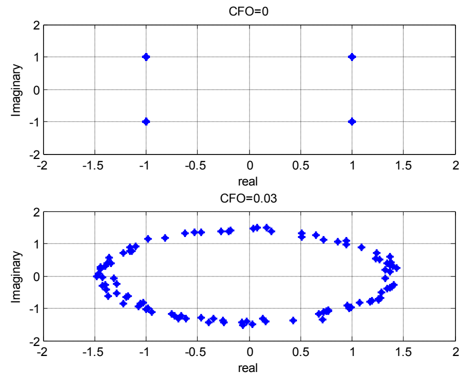

Figure 2. Received signals affected by CFO

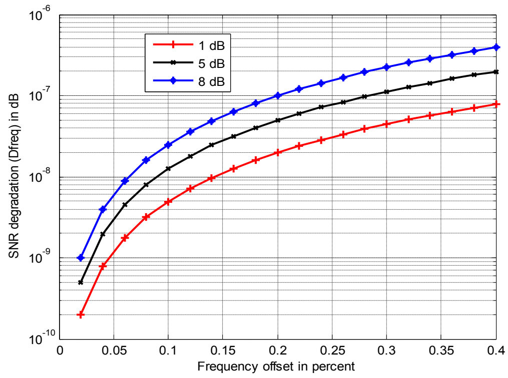

Figure 3. SNR degradation due to CFO

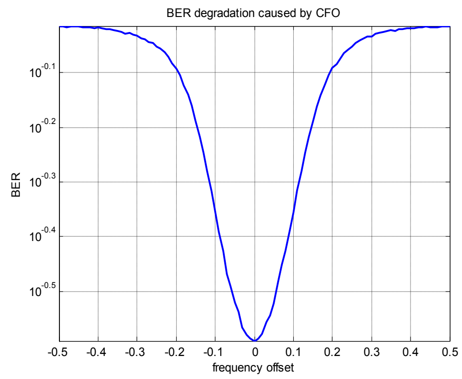

Figure 4. BER degradation caused by CFO

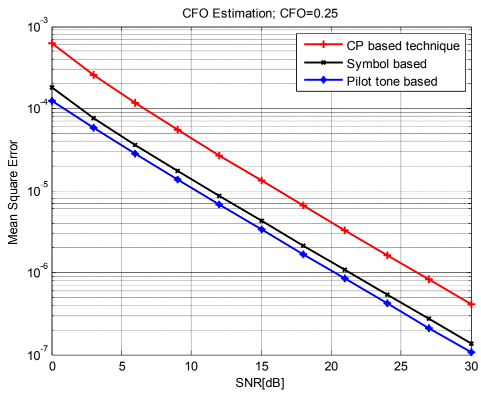

Figure 5. Comparison between CFO Estimation methods

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